Quark Masses and Mixings with Hierarchical Friedberg-Lee Symmetry

Takeshi Araki* and C. Q. Geng†

Department of Physics, National Tsing Hua University, Hsinchu, Taiwan 300

Abstract

We consider the Friedberg-Lee symmetry for the quark sector and show that the symmetry closely relates to both quark masses and mixing angles. We also extend our scheme to the fourth generation quark model and find the relation $|V_{tb'}| \simeq |V_{t'b}| \simeq m_b/m_{b'} < \lambda^2$ with $\lambda \simeq 0.22$ for $m_b = 4.2$ GeV and $m_{b'} > 199$ GeV.

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* araki@phys.nthu.edu.tw
† geng@phys.nthu.edu.tw

I. INTRODUCTION

Although the standard model (SM) is a very successful theory, there are still some mysteries and problems. One of them is the flavor structure of fermions. We currently know that quarks mix with each other through the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1] and that there is a hierarchy among the three mixing angles of the CKM matrix: $\theta_{12} \simeq \lambda \simeq 0.22$, $\theta_{23} \simeq \lambda^2$ and $\theta_{13} \simeq \lambda^4$ [2]. Yet another hierarchy exists among quark masses and it can be expressed in terms of λ as $m_u/m_c \simeq m_c/m_t \simeq \lambda^4$ and $m_d/m_s \simeq m_s/m_b \simeq \lambda^2$ [3]. In particular, θ_{12} and $\sqrt{m_d/m_s}$ surprisingly coincide with each other. However, the origin of the hierarchies is still unclear. This is because the Yukawa sector in the SM contains a huge number of unknown parameters. Consequently, fermion masses and mixing angles remain free parameters in the SM. Thus, it is interesting to extend the SM with a family symmetry. In Ref. [4], authors introduce a family $U(1)$ symmetry and try to explain the quark mass hierarchies and small mixing angles via hierarchically suppressed nonrenormalizable Yukawa interactions. A finite group could also be the prime candidate of a family symmetry, which can reduce the number of parameters in the Yukawa sector and make the theory predictive [5–9].

Recently, Friedberg and Lee proposed a (hidden) translational family symmetry called Friedberg-Lee (FL) symmetry [10] and tried to relate quark mixing angles with the symmetry. A more detailed analysis is given in Ref. [11] and an application to the fourth generation quark model is discussed in Ref. [12]. Furthermore, many attempts for the lepton sector are performed in Refs. [13, 14], some possible origins of the symmetry have been discussed in Ref. [15], and a new spontaneous CP violation mechanism with the symmetry is considered in Ref. [16]. Moreover, one of the motivations to consider the FL symmetry is that the smallness of up- and down-quark masses as well as the electron mass can be naturally understood as it always makes one of three generation fermions massless [10, 14]. However, the original approach [10] with the FL symmetry cannot explicitly reveal the hierarchies of quark masses and mixing angles, which are treated as input parameters. In this paper, we extend the discussion to connect both quark masses and mixing angles with the FL symmetry. In particular, as will be explained in the next section, we propose the FL symmetry which translates the three generation quarks hierarchically and also show that the hierarchical patterns can be responsible for the quark

flavor structures mentioned above.

This paper is organized as follows. In Sec. II, we present the hierarchical FL symmetry. In Sec. III, we break both FL and CP symmetries in order to generate light quark masses and the CP violating Dirac phase in the CKM matrix. In Sec. IV, we extend our model to the fourth generation case. We summarize our results in Sec. V.

II. HIERARCHICAL FRIEDBERG-LEE SYMMETRY

We start our discussion with the Lagrangian of the quark mass terms

$$- \mathcal{L} = M^d_{ij}\overline{d_i}d_j + M^u_{ij}\overline{u_i}u_j \ , \tag{1}$$

where $M^{u,d}$ are up- and down-type quark mass matrices, which are assumed to be symmetric. The subscripts $i, j = 1, ..., 3$ stand for the family indices. For the Lagrangian, we impose the FL symmetry [10]

$$q_i \to q_i + (1, \eta_q, \eta_q\xi_q)^T z_q, \tag{2}$$

where η_q and ξ_q are c-numbers, z_q is a global Grassmann parameter and $q = d, u$. Because of the symmetry, the quark mass matrices take the form

$$M^q = \begin{pmatrix} B_q\eta_q^2 + C_q & -B_q\eta_q & -C_q/(\eta_q\xi_q) \\ -B_q\eta_q & A_q\xi_q^2 + B_q & -A_q\xi_q \\ -C_q/(\eta_q\xi_q) & -A_q\xi_q & A_q + C_q/(\eta_q\xi_q)^2 \end{pmatrix}, \tag{3}$$

where we have assumed that A_q, B_q, C_q, η_q, and ξ_q are real. Hence, the theory is CP conserving. Also, the up and down quarks are massless because of the FL symmetry. In the next section, we will insert phase factors into the mass matrices to generate the light quark masses and CP violation.

In order to make our point more clear, here we redefine the parameters as follows:

$$C_u \to C_u\eta_u^2\xi_u, \tag{4}$$

$$C_d \to C_d\eta_d^2\xi_d, \ A_d \to A_d/\xi_d. \tag{5}$$

Consequently, the up- and down-type quark mass matrices become

$$M^u = \begin{pmatrix} (B_u + C_u\xi_u)\eta_u^2 & -B_u\eta_u & -C_u\eta_u \\ -B_u\eta_u & A_u\xi_u^2 + B_u & -A_u\xi_u \\ -C_u\eta_u & -A_u\xi_u & A_u + C_u/\xi_u \end{pmatrix} \tag{6}$$

and

$$M^d = \begin{pmatrix} (B_d + C_d\xi_d)\eta_d^2 & -B_d\eta_d & -C_d\eta_d \\ -B_d\eta_d & A_d\xi_d + B_d & -A_d \\ -C_d\eta_d & -A_d & (A_d + C_d)/\xi_d \end{pmatrix}, \tag{7}$$

respectively. In this basis, we impose

$$A_q \simeq B_q \simeq C_q \quad \text{and} \quad \eta_q, \xi_q \ll 1 \ . \tag{8}$$

We note that with Eq. (8), the down-type quark mass matrix becomes similar to the hybrid texture discussed in Ref. [17]. On the other hand, the up-type one is almost diagonal. Since $M^{u,d}$ are real and symmetric matrices, they can be diagonalized by real orthogonal matrices: $V_q^T M^q V_q = \mathrm{diag}(m_{q1}, m_{q2}, m_{q3})$. With Eq. (8), V_q can approximately be written as

$$V_u \simeq \begin{pmatrix} 1 & -\eta_u & 0 \\ \eta_u & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{9}$$

with $m_u/m_c \simeq \eta_u^2$ and $m_c/m_t \simeq |\xi_u|$ for the up-quark sector, and

$$V_d \simeq \begin{pmatrix} 1 & -\eta_d & -\frac{1}{2}\eta_d\xi_d \\ \eta_d & 1 & -\frac{1}{2}\xi_d \\ \eta_d\xi_d & \frac{1}{2}\xi_d & 1 \end{pmatrix} \tag{10}$$

with $m_d/m_s \simeq \eta_d^2$ and $m_s/m_b \simeq 1/2|\xi_d|$ for the down-quark sector, respectively. Note that the factor 1/2 in Eq. (10) arises from the $\{33\}$ element of Eq. (7). From the mass ratios, we can deduce that

$$|\eta_u| \simeq \lambda^2, \quad |\xi_u| \simeq \lambda^4, \tag{11}$$

$$|\eta_d| \simeq \lambda, \quad |\xi_d| \simeq \lambda^2, \tag{12}$$

with $\lambda \simeq 0.22$. Therefore, in what follows, we consider the following hierarchical FL translation:

$$u_i \to u_i + (1, -\lambda^2, -\lambda^6)^T z_u \ , \tag{13}$$

$$d_i \to d_i + (1, \lambda, \lambda^3)^T z_d \ . \tag{14}$$

The minus signs in the up sector come from η_u which is negative to reproduce realistic CKM elements. We note that although the up and down quarks are massless at this stage, in the above approximation, we remain m_u and m_d to be nonzero in order to determine the order of η_q. The parameters $\eta_u = -\lambda^2$ and $\eta_d = \lambda$ would be the origin of $m_u/m_c \sim \lambda^4$ and $m_d/m_s \sim \lambda^2$, respectively, when we introduce the symmetry breaking terms.

On the other hand, the parameters η_q and ξ_q can also be the origin of the tiny mixing angles of the CKM matrix. For instance, three elements of the CKM matrix can be estimated as

$$\begin{aligned}
&|V_{us}| \simeq |-\eta_d + \eta_u| \simeq 1.25\lambda, \\
&|V_{ub}| \simeq 0.5|-\eta_d\xi_d - \eta_u\xi_d| \simeq 1.5\lambda^4, \\
&|V_{cb}| \simeq 0.5|\xi_d| \simeq 0.5\lambda^2,
\end{aligned} \tag{15}$$

where we have used $\lambda \simeq 4\lambda^2$. These results well coincide with the experimental values $\theta_{12} \simeq \lambda$, $\theta_{23} \simeq \lambda^2$, and $\theta_{13} \simeq \lambda^4$ mentioned in the Introduction. Namely, in our model, the flavor structures in the quark sector are explained by the hierarchical patterns of the symmetry. Hence, it is also easy to establish the relations between the masses and mixing angles, such as

$$\sqrt{m_d/m_s} \sim |V_{us}| \tag{16}$$

via η_d, and

$$m_s/m_b \sim |V_{cb}| \tag{17}$$

via ξ_d, respectively.

III. FL SYMMETRY BREAKING AND PARAMETER FITTING

In order to generate the light quark masses, the FL symmetry must be broken. Since we do not know the origin of the breaking, there may exist many possible ways to break the symmetry. Here, we aim at a minimal scheme and put phase factors into the mass matrices to break the FL and CP symmetries simultaneously as discussed in Ref. [10].

That is, we replace Eqs. (6) and (7) as

$$M^u = \begin{pmatrix} (B_u + C_u\xi_u)\eta_u^2 & -B_u\eta_u\ e^{i\theta_1} & -C_u\eta_u \\ -B_u\eta_u\ e^{i\theta_1} & A_u\xi_u^2 + B_u & -A_u\xi_u\ e^{i\theta_2} \\ -C_u\eta_u & -A_u\xi_u\ e^{i\theta_2} & A_u + C_u/\xi_u \end{pmatrix}, \tag{18}$$

$$M^d = \begin{pmatrix} (B_d + C_d\xi_d)\eta_d^2 & -B_d\eta_d\ e^{i\phi_1} & -C_d\eta_d \\ -B_d\eta_d\ e^{i\phi_1} & A_d\xi_d + B_d & -A_d\ e^{i\phi_2} \\ -C_d\eta_d & -A_d\ e^{i\phi_2} & (A_d + C_d)/\xi_d \end{pmatrix}, \tag{19}$$

respectively. Note that phase factors are added into $\{12\}, \{21\}, \{23\}$ and $\{32\}$ elements for each matrix. The phases θ_1 and ϕ_1 are responsible for the up- and down-quark masses. If we regard $(e^{i\theta_1} - 1)$ and $(e^{i\phi_1} - 1)$ as perturbations, we obtain

$$\frac{m_u}{m_c} \simeq \left|2\eta_u^2(e^{i\theta_1} - 1)\right|, \quad \frac{m_d}{m_s} \simeq \left|2\eta_d^2(e^{i\phi_1} - 1)\right| \ , \tag{20}$$

as we expected. The phases θ_2 and ϕ_2 are added to account for the CP violating Dirac phase in the CKM matrix.

In the following, we will present our numerical analysis to illustrate our result. In our calculation, without loss of generality, we will normalize Eqs. (18) and (19) with $B_{u,d} = 1$. We will also ignore the terms associated with A_u because they are suppressed by $\xi_u = \lambda^4$. As a result, the model has three real parameters: A_d and $C_{u,d}$, and three CP violating phases: θ_1 and $\phi_{1,2}$. For these six parameters, we use the following experimental values from the Particle Data Group (PDG) [18]:

$$m_u/m_c = (0.112 - 0.284) \times 10^{-2},\ m_c/m_t = (0.669 - 0.792) \times 10^{-2}, \tag{21}$$

$$m_s/m_d = 17 - 22,\ m_s/m_b = (0.160 - 0.315) \times 10^{-1}, \tag{22}$$

$$|V_{us}| = 0.2236 - 0.2274,\ |V_{cb}| = 0.0401 - 0.0423, \tag{23}$$

as input parameters. Figure 1 shows the allowed region in the $|V_{us}| - |V_{td}/V_{ts}|$ plane with the bounds [18]

$$|V_{ub}| = (3.57 - 4.29) \times 10^{-3}, \quad |V_{td}/V_{ts}| = 0.202 - 0.216. \tag{24}$$

One can easily see from Fig. 1 that our results on $|V_{us}|$ and $|V_{td}/V_{ts}|$ are consistent with

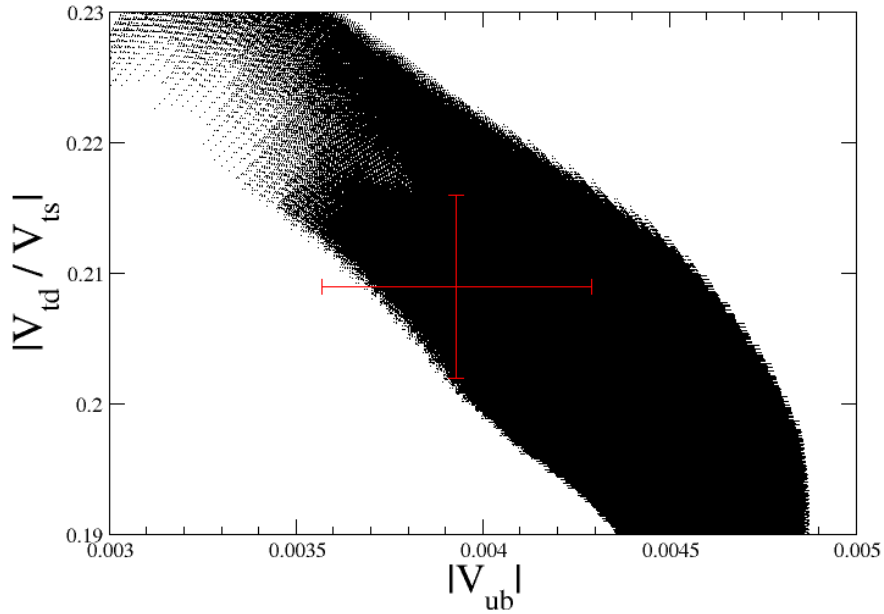

FIG. 1: The allowed region in the $|V_{ub}| - |V_{td}/V_{ts}|$ plane, where the experimental values from the PDG [18] are also plotted as the cross lines.

the experimental data. Here, we would like to emphasize that the flavor structures of the quark sector are mainly originated from the hierarchical FL symmetry even though the parameters A_d and $C_{u,d}$ are not completely fixed. At the best fit point, we have

$$C_u \simeq 0.4, \;\; A_d \simeq 1.6, \;\; C_d \simeq 0.34 \tag{25}$$

with $B_{u,d} = 1$, which fill the gap between the exact values and expected ones estimated in the previous section.

IV. IMPLICATIONS FOR FOURTH GENERATION QUARK MODEL

In our scheme, quark masses and mixing angles are closely related with each other, such as those in Eqs. (16) and (17). So, it should be interesting to extend our scheme to the fourth generation quark model and see whether there exist relations between the fourth generation quark masses and their mixing angles.

We extend the FL symmetry as

$$q_i \to q_i + (1, \eta_q, \eta_q\xi_q, \eta_q\xi_q\rho_q)^T z_q \ , \tag{26}$$

and the mass matrices Eqs. (6) and (7) as

$$M^u \ = \begin{pmatrix} (B_u + C_u\xi_u + D_u\xi_u\rho_u)\eta_u^2 & -B_u\eta_u & -C_u\eta_u & -D_u\eta_u \\ -B_u\eta_u & A_u\xi_u^2 + B_u + E_u\xi_u^2\rho_u^2 & -A_u\xi_u & -E_u\xi_u\rho_u \\ -C_u\eta_u & -A_u\xi_u & A_u + \frac{C_u}{\xi_u} + F_u\xi_u^2\rho_u^3 & -F_u\xi_u^2\rho_u^2 \\ -D_u\eta_u & -E_u\xi_u\rho_u & -F_u\xi_u^2\rho_u^2 & \frac{D_u}{\xi_u\rho_u} + E_u + F_u\xi_u^2\rho_u \end{pmatrix} \tag{27}$$

and

$$M^d \ = \begin{pmatrix} (B_d + C_d\xi_d + D_d\xi_d\rho_d)\eta_d^2 & -B_d\eta_d & -C_d\eta_d & -D_d\eta_d \\ -B_d\eta_d & A_d\xi_d + B_d + E_d\xi_d\rho_d & -A_d & -E_d \\ -C_d\eta_d & -A_d & \frac{A_d+C_d}{\xi_d} + F_d\frac{\rho_d}{\xi_d} & -\frac{F_d}{\xi_d} \\ -D_d\eta_d & -E_d & -\frac{F_d}{\xi_d} & \frac{D_d+E_d+F_d}{\xi_d\rho_d} \end{pmatrix} , \tag{28}$$

respectively. Here, we again assume that

$$A_q \simeq B_q \simeq C_q \simeq D_q \simeq E_q \simeq F_q \ \text{ and } \ \eta_q, \xi_q, \rho_q \ll 1 \ . \tag{29}$$

Note that we extend the model so that the mass matrices keep the features mentioned just behind Eq. (8). Since a detailed analysis goes beyond the purpose of the paper, we would like to roughly study and try to figure out their implications. Both Eqs. (27) and (28) are diagonalized by real orthogonal matrices. One can easily find that ρ_q determine the mass ratios of the fourth and third generation quarks:

$$\frac{m_t}{m_{t'}} \sim \rho_u \ , \quad \frac{m_b}{m_{b'}} \sim \rho_d \ , \tag{30}$$

where t' and b' indicate the fourth generation quarks, and all coefficients are omitted. The fourth generation quark mixings with the other three generations can also be estimated as

$$\begin{aligned} &|V_{ub'}| \simeq |V_{t'd}| \simeq |\eta_d\xi_d\rho_d| = \lambda^3|\rho_d| \ , \\ &|V_{cb'}| \simeq |V_{t's}| \simeq |\xi_d\rho_d| = \lambda^2|\rho_d| \ , \\ &|V_{tb'}| \simeq |V_{t'b}| \simeq |\rho_d| \ , \end{aligned} \tag{31}$$

where we have used $\eta_d = \lambda$ and $\xi_d = \lambda^2$. As one can see, the fourth generation quark mixing angles are directly related to the mass ration $m_b/m_{b'}$ via ρ_d. We note that the contribution of ρ_u to the mixing angles is negligibly small compared with that of ρ_d. To illustrate our implications, we substitute the lower bound of $m_{b'}$, i.e., $m_{b'} > 199$ GeV [18], and the central value $m_b = 4.2$ GeV [18], into Eq. (30). Then we get $\rho_d < \lambda^2$ and

$$|V_{ub'}| \simeq |V_{t'd}| < \lambda^5 \ , \tag{32}$$

$$|V_{cb'}| \simeq |V_{t's}| < \lambda^4 \ , \tag{33}$$

$$|V_{tb'}| \simeq |V_{t'b}| < \lambda^2 \ . \tag{34}$$

It is interesting to see that $|V_{tb'}|$ and $|V_{t'b}|$ can be large, which could be measurable at the upcoming experiments like the LHC.

Finally, we briefly comment about CP violation. Although, in general, the discussion of CP violation in the fourth generation model is very complicated, in the chiral limit of $m_{u,d,s,c} = 0$, CP violation is described by only one simple quantity [19]

$$J_4 = \mathrm{Im}[V_{tb} V^*_{t'b} V_{t'b'} V^*_{tb'}] \ . \tag{35}$$

As discussed in Ref. [12], the quantity may provide us useful information about the size of CP violation. By introducing the same phase factors discussed in Sec. III, we find that $J_4 \simeq 10^{-7}$. Unfortunately, the result is much smaller than the SM Jarlskog invariant parameter $J_{SM} \simeq 10^{-5}$. Hence, the fourth generation model, in our scheme, cannot be the main source of the CP violating phenomena, such as the baryon asymmetry of the Universe [20].

V. SUMMARY

We have studied the quark masses and mixing angles with the hierarchical FL symmetry. We have shown that the symmetry can explain the hierarchies in the quark masses and mixing angles at the same time. As a result, the masses and mixing angles are closely related with each other in our model. To generate light quark masses and CP violation, we have introduced phase factors into the mass matrices, and then demonstrated that our model can reproduce all experimental values. We have also extended our scheme to

the fourth generation quark model and found the relation $|V_{tb'}| \simeq |V_{t'b}| \simeq m_b/m_{b'} < \lambda^2$ for $m_b = 4.2$ GeV and $m_{b'} > 199$ GeV. We have speculated that $|V_{tb'}|$ and $|V_{t'b}|$ could be measurable at the upcoming experiments like the LHC in our extended fourth generation quark model.

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